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非线性广义半马尔可夫跳跃系统的H_∞控制

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摘　要

探讨一类非线性广义半Markov跳跃系统的随机稳定性和H_∞控制问题。设计新型的Lyapunov-Krasovkii泛函(LKF)，用于减少冗余决策变量。同时，引入参数依赖的互凸矩阵不等式(PDRCMI)来降低保守性，保证了非线性广义Markov跳跃系统渐进稳定并满足性能，最后，通过数值算例验证了所得方法的有效性。

关键词

广义半马尔可夫跳跃系统，非线性，参数相关互凸矩阵不等式，H_∞控制

H_∞ Control of Nonlinear Singular Semi-Markov Jump Systems

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Abstract

The stochastic stability of a class of nonlinear singular semi-Markov jumping systems and H_∞ control problems are discussed. A new type of Lyapunov-Krasovkii functional (LKF) is designed to reduce redundant decision variables. At the same time, the parameter-dependent Convex Matrix Inequality (PDRCMI) is introduced to reduce the conservatism, which ensures the asymptotic stability and satisfies the performance of the nonlinear singular Markov jumping system, and finally, the effectiveness of the proposed method is verified by numerical examples.

Keywords

Singular Semi-Markov Jump System, Nonlinearity, Parameter-Dependent Reciprocally Convex

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Matrix Inequality, H_∞ Control

1. 引言

广义系统因其广泛应用于电力的系统、航空航天工程、社会经济系统、化学过程、生物系统、网络分析、飞机控制系统、电力网络、太阳能热中央接收器、机器人机械手系统等[1]-[3]领域而引起了广泛的兴趣。几十年来，稳定性问题得到了广泛的研究[1]-[3]。然而，在这些实际系统的参数和结构中，一些现象，如子系统互连的修改、突变环境干扰、组件故障或维修等原因，可能导致系统结构，参数发生随机变化。而幸运的是，这些变化可以用马尔可夫跳跃系统(MJSs)适当地描述，并且每种操作模式都与一个动态系统关联，其中模型转换在一个马尔可夫过程的控制下。因此，学术界和工业界的大量关注都集中在广义马尔可夫跳跃系统(SMJSs)的研究上。

MJSs 作为一种特殊的随机系统，由于其理论意义已经在网络系统控制、能源、制造和经济系统[1]-[6]中得到了广泛的研究。许多问题已经解决，取得良好的结果，例如，H_∞控制滤波的不确定时滞 MJSs [2]，具有部分未知转移速率的 MJSs 的稳定性分析[3]，不定二次最优控制问题[4]，基于分散二维马尔可夫跳跃系统的故障检测[7]。

2. 新判据

2.1. 模型介绍与假设

本文采用以下记号本文考虑下述不确定广义半马尔可夫跳跃系统:

$$\begin{cases} E\dot{x}(t)=\left(A(r_t)+\Delta A(r_t)\right)x(t)+\left(A_d(r_t)+\Delta A_d(r_t)\right)x\left(t-d(t)\right)+B_w(r_t)\varpi(t) \\ \qquad +B(r_t)\left(u(t)+f\left(t,x(t),r(t)\right)\right)+B_d(r_t)\left(u\left(t-d(t)\right)+f\left(t,x\left(t-d(t)\right),r(t)\right)\right) \\ z(t)=C(r_t)x(t)+C_d(r_t)x\left(t-d(t)\right)+D(r_t)w(t) \end{cases} \tag{2.1}$$

其中 $x(t)\in\mathbb{R}^n$ 为系统状态向量，$u(t)\in\mathbb{R}^m$ 为控制输入，$z(t)\in\mathbb{R}^p$ 为控制输出，$w(t)\in\mathbb{R}^q$ 为外部扰动，矩阵 $E\in\mathbb{R}^{n\times n}$ 可能是奇异矩阵，$Rank(E)=r\le n$，$f\left(t,x(t)\right)$ 为非线性函数。$A(r_t)$，$A_d(r_t)$，$B(r_t)$，$B_d(r_t)$，$B_w(r_t)$，$C(r_t)$，$C_d(r_t)$ 和 $D(r_t)$ 是适当维度的已知常数矩阵。$\Delta A(r_t)$ 和 $\Delta A_d(r_t)$ 是未知时变矩阵。时间延迟 $d(t)$ 满足 $0\le d_1\le d(t)\le d_2$，$h_1\le\dot{d}(t)\le h_2$，$\forall t\ge 0$，d_1,d_2,h_1,h_2 是给定常数边界，同时定义 $0<d_{12}=d_2-d_1$。模态 $\{r_t,t\ge 0\}$ 是连续时间半马尔可夫过程，该过程在有限集中取值 $S=\{1,2,\cdots,s\}$ [1]，转移速率矩阵 $\Pi=\{\pi_{ij}\}$ 如下定义：

$$\mathrm{P}\left\{r(t+\Delta)=j\,\middle|\,r(t)=i\right\}=\begin{cases}\pi_{ij}\Delta+o(\Delta), & i\ne j,\\ 1+\pi_{ii}\Delta+o(\Delta), & i=j,\end{cases}$$

这里 $\Delta>0$，$\lim\limits_{\Delta\to 0}\dfrac{o(\Delta)}{\Delta}=0$ 及 $\pi_{ij}(h)\ge 0(i,j\in S,i\ne j)$，指将 i 从 t 的模态转变到 $t+\Delta$ 的 j 模态，$\forall i\in S$ 都有 $\pi_{ij}=-\sum\limits_{j=1,j\ne i}^{s}\pi_{ij}$。为了便于表达定义：

$$A(r_t)=A_i,A_d(r_t)=A_{di},B(r_t)=B_i,B_d(r_t)=B_{di},B_w(r_t)=B_{wi},C(r_t)=C_i,C_d(r_t)=C_{di},D(r_t)=D_i$$

$$\Delta A(r_t)=\Delta A_i,\Delta A_d(r_t)=\Delta A_{di},f(t,x(t),r(t))=f(t),f(t,x(t-d(t)),r(t))=f(t-d(t)).$$

为了简化系统的形式，我们可以将系统(2.1)重写如下：

$$\begin{cases} E\dot{x}(t)=(A_i+\Delta A_i)x(t)+(A_{di}+\Delta A_{di})x(t-d(t))+B_i(u(t)+f(t)) \\ \qquad +B_{di}(u(t-d(t))+f(t-d(t)))+B_{wi}\varpi(t) \\ Z(t)=C_ix(t)+C_{di}x(t-d(t))+D_iw(t) \end{cases} \tag{2.2}$$

假设转移速率矩阵 $\Pi\triangleq(\pi_{ij})$ 通常不确定，我们为系统(2.1)计算转移速率矩阵，如下所示：

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta\pi_{11} & ? & \hat{\pi}_{13}+\Delta\pi_{13} & \cdots & ? \\ ? & ? & \hat{\pi}_{23}+\Delta\pi_{23} & \cdots & \hat{\pi}_{2s}+\Delta\pi_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \hat{\pi}_{s2}+\Delta\pi_{s2} & ? & \cdots & \hat{\pi}_{ss}+\Delta\pi_{ss} \end{bmatrix} \tag{2.3}$$

其中 $\hat{\pi}_{ij}$ 和 $\Delta\pi_{ij}\in[-\delta_{ij},\delta_{ij}](\delta_{ij}\ge 0)_{ij}\in[-\delta_{ij},\delta_{ij}](\delta_{ij}\ge 0)$ 分别代表估计值和不确定转移速率 π_{ij} 的估计误差。$\hat{\pi}_{ij}$ 和 δ_{ij} 为已知，“?”是完全未知的。对任意 $i\in S$，集合 U^i 表示 $U^i=U_k^i\cup U_{uk}^i$。其中 $U_k^i\triangleq\{j:\pi_{ij}\text{为已知},j\in S\}$，$U_{uk}^i\triangleq\{j:\pi_{ij}\text{为未知},j\in S\}$。此外，若 $U_k^i\ne\varnothing$，它可以被描述为 $U_k^i=\{k_1^i,k_2^i,\cdots,k_{mi}^i\}$，其中 $k_{mi}^i\in S^+$ 表示矩阵 Π 的第 i 排的第 m 个已知边界。

备注 2.1 无论是有界不确定的 TR(BUTR)或部分未知 TR(PUTR)模型均不如上述不确定转移速率的描述那么宽泛。重写了以下两个不确定的模型：

$$\begin{bmatrix} \hat{\pi}_{11}+\Delta_{11} & \hat{\pi}_{12}+\Delta_{12} & \cdots & \hat{\pi}_{1s}+\Delta_{1s} \\ \hat{\pi}_{21}+\Delta_{21} & \hat{\pi}_{22}+\Delta_{22} & \cdots & \hat{\pi}_{2s}+\Delta_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{s1}+\Delta_{s1} & \hat{\pi}_{s2}+\Delta_{s2} & \cdots & \hat{\pi}_{ss}+\Delta_{ss} \end{bmatrix} \tag{2.4}$$

其中 $\hat{\pi}_{ij}-\delta_{ij}\ge 0(\forall j\in S,j\ne i)$，$\hat{\pi}_{ii}=-\sum_{j=1,j\ne i}^{s}\hat{\pi}_{ij}$ 和 $\delta_{ii}=-\sum_{j=1,j\ne i}^{s}\delta_{ij}$。

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & \cdots & ? \\ ? & ? & \pi_{23} & \cdots & \pi_{2s} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ ? & \pi_{s2} & ? & \cdots & \pi_{ss} \end{bmatrix} \tag{2.5}$$

显然，当 $U_k^i\ne\varnothing$，$\forall i\in S$ 时，GUTR 模型(2.3)简化为 BUTR 模型(2.5)，若 $\delta_{ij}=0$，$\forall i\in S$，$\forall j\in U_k^i$ 则简化为 PUTR 模型(2.5)。显然，BUTR 或 PUTR 模型不如 GUTR 模型(2.3)通用，这意味着它更实用。

假设已知 TR 的估计值如下定义：

假设 2.1 若 $U_k^i=S$，则 $\hat{\pi}_{ij}-\delta_{ij}\ge 0$，$(\forall j\in S,j\ne i)$，$\hat{\pi}_{ii}=-\sum_{j=1,j\ne i}^{s}\hat{\pi}_{ij}\le 0$，$\delta_{ii}=\sum_{j=1,j\ne i}^{s}\delta_{ij}>0$；

假设 2.2 若 $U_k^i\ne S$ 且 $i\in U_k^i$，则 $\hat{\pi}_{ij}-\delta_{ij}\ge 0$，$(\forall j\in U_k^i,j\ne i)$，$\hat{\pi}_{ii}+\delta_{ii}\le 0$，$\sum_{j\in U_k^i}\hat{\pi}_{ij}\le 0$；

假设 2.3 若 $U_k^i\ne S$ 且 $i\in U_k^i$，则 $\hat{\pi}_{ij}-\delta_{ij}\ge 0$，$(\forall j\in U_k^i)$。

这三个假设是基于 TR 特征而得出的，由此可以推断它们具有合理性。

$$\left(e.g.\ \pi_{ij}\ge 0(\forall j\in S,j\ne i)\text{ and }\pi_{ii}=-\sum_{j=1,j\ne i}^{s}\pi_{ij}\right).$$

假设 2.4 在本文中所涉及的不确定性属于有界范数的，假设如下：

$$\left[\Delta A_i\ \Delta A_{di}\right]=M_iF_i(t)\left[N_i\ N_{di}\right],\forall i\in S$$

已知的常数矩阵 M_i，N_i，N_{di} 具有适当的维度，使 $F_i(t)$ 得满足 $F_i^{\mathrm{T}}(t)F_i(t)\le I$，$i\in S$。

假设 2.5 非线性函数 $f(t,\xi(t))$ 满足

$$\begin{cases} f^{\mathrm{T}}(t)f(t)\le x^{\mathrm{T}}(t)T^2x(t) \\ f^{\mathrm{T}}(t-d(t))f(t-d(t))\le x^{\mathrm{T}}(t-d(t))T^2x(t-d(t)) \end{cases}$$ 其中 $T=diag\{\sigma_1,\sigma_2,\cdots,\sigma_s\}$。

定义 2.1 [7]若 $\det(s\hat{E}-\hat{A}_i)$ 对任意 $r_t=i\in\mathbf{S}$ 均不为 0，则系统(2.2)正则。

1) 若对任意 $r_t=i\in\mathbf{S}$ 有 $\deg\left(\det\left(s\hat{E}-\hat{A}_i\right)\right)=rank\left(\hat{E}\right)$，则系统(2.2)被称为无脉冲。

2) 当 $w(t)=0$，$u(t)=0$ 时且存在标量 $M(r_0,\varphi(t))>0$ 使得 $\lim_{t\to\infty}\varepsilon\left\{\int_0^\infty\|x(t)\|^2\mathrm{d}t\,\middle|\,r_0,x(t)=\varphi(t),t\in[-h,0]\right\}\le M(r_0,\varphi(t))$，则(2.2)随机稳定的。

3) 当 $w(t)=0$，$u(t)=0$ 时，若系统(2.2)是正则的、无脉冲的，随机稳定的，则称它是随机容许的。

定义 2.2 对于给定的标量 $\gamma>0$，系统(2.2)是随机容许的并满足 H_∞性能 γ，如果它在 $w(t)=0$ 及零初始条件下是随机容许的，非零的 $w(t)\in L_2[0,\infty)$，满足以下条件：

$$\varepsilon\left\{\int_0^\infty\left(z^{\mathrm{T}}(t)z(t)-\lambda^2w^{\mathrm{T}}(t)w(t)\right)\mathrm{d}t\right\}<0.$$

引理 2.1 设 H 和 G 是具有适当维数的实数矩阵，并且 $F_i^{\mathrm{T}}(t)F_i(t)\le I$。以下不等式适用于任何标量 $\lambda>0$：

1) $HF_i(t)G^{\mathrm{T}}+GF_i^{\mathrm{T}}(t)H^{\mathrm{T}}\le\lambda GG^{\mathrm{T}}+\lambda^{-1}HH^{\mathrm{T}}$，2) $\pm2H^{\mathrm{T}}G\le H^{\mathrm{T}}H+G^{\mathrm{T}}G$。

引理 2.2 设 $C\in R^{n\times n}$，$B\in R^{m\times m}$ 为正定矩阵，当且仅当 $A>B^{\mathrm{T}}C^{-1}B$，则对 $\forall A\in R^{m\times m}$ 有

$$F=\begin{pmatrix}A & B^{\mathrm{T}}\\ B & C\end{pmatrix}>0$$

2.2. 主要结果

对于给定的标量 $0\le d_1\le d_2$，h_1，h_2，$\varepsilon_1\ge0$，$\varepsilon_2\ge0$ 和 $\gamma>0$，如果存在矩阵 $P_i>0$，$R_{1i}>0$，$R_{2i}>0$，$R_{3i}>0$，$Q_1>0$，$Q_2>0$，$S_1>0$，$S_2>0$，$Z_1>0$，$Z_2>0$，Y_1，Y_2 使得以下不等式对任意 $i\in I$ 成立，则系统渐进稳定的并具有相应的 H_∞衰减指数 γ

$$E^{\mathrm{T}}P_i=P_i^{\mathrm{T}}E>0,$$

$$\Omega^{[m]}=\begin{bmatrix}E^{[m]} & E_{12}\\ & E_{22}\end{bmatrix}<0,\ m=1,2,3,4,$$

$$\sum_{j=1}^N\pi_{ij}R_{1j}-Q_1\le0,\ \sum_{j=1}^N\pi_{ij}R_{3j}-Q_2\le0,$$

$$\sum_{j=1}^N\pi_{ij}R_{1j}-Q_1\le0,\ \sum_{j=1}^N\pi_{ij}R_{3j}-Q_2\le0,\ R_{2i}-R_{3i}\le0,\ \Phi_1>0,\ \Phi_2>0,$$

$$E\left(d(t),\dot{d}(t)\right)=E_{1i\mu|d(t),\dot{d}(t)}+E_2+E_3,$$

$$E_{1i\mu|d(t),\dot{d}(t)}=\Xi+\Lambda_1^{\mathrm{T}}\left(\sum_{j=1}^N\pi_{ij}P_j\right)\Lambda_1+x^{\mathrm{T}}(t)E^{\mathrm{T}}\sum_{j=1}^N\pi_{ij}P_jx(t),$$

$$E_{1i\mu|d(t),\dot{d}(t)} = sym\left(\Lambda_1^{\mathrm{T}} P_i \Lambda_2\right) + \Lambda_1^{\mathrm{T}}\left(\sum_{j=1}^{N} \pi_{ij} P_j\right)\Lambda_1 + e_1^{\mathrm{T}}\left(R_{1i} + R_{3i} + d_1 Q_1 + d_2 Q_2 + \Theta_1\right) e_1$$
$$+ e_2^{\mathrm{T}}\left(\lambda_{i6}^{-1} T^2 - R_{1i}\right) e_2 - e_3^{\mathrm{T}} R_{2i} e_3 + e_5^{\mathrm{T}}\left(d_2^2 S_1 + d_{12}^2 S_2 + \frac{1}{2} d_1^2 \left(Z_1 + Z_2\right)\right) e_5$$
$$- \Lambda_6^{\mathrm{T}} \bar{Z}_1 \Lambda_6 - \Lambda_7^{\mathrm{T}} \bar{Z}_2 \Lambda_7 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j=1}^{N} \pi_{ij} P_j x(t) + mt e_4^{\mathrm{T}}\left(R_{2i} - R_{3i}\right) e_4 + e_1^{\mathrm{T}} P_i \Theta_2 e_2,$$

$$\Xi = sym\left(\Lambda_1^{\mathrm{T}} P_i \Lambda_2\right) + e_1^{\mathrm{T}}\left(R_{1i} + R_{3i} + d_1 Q_1 + d_2 Q_2 + \Theta_1\right) e_1 + e_2^{\mathrm{T}}\left(\lambda_{i6}^{-1} T^2 - R_{1i}\right) e_2 - e_3^{\mathrm{T}} R_{2i} e_3$$
$$+ mt e_4^{\mathrm{T}}\left(R_{2i} - R_{3i}\right) e_4 + e_1^{\mathrm{T}} P_i \Theta_2 e_2 + e_5^{\mathrm{T}}\left(d_2^2 S_1 + d_{12}^2 S_2 + \frac{1}{2} d_1^2\left(Z_1 + Z_2\right)\right) e_5 - \Lambda_6^{\mathrm{T}} \bar{Z}_1 \Lambda_6 - \Lambda_7^{\mathrm{T}} \bar{Z}_2 \Lambda_7,$$

$$E_2 = -\Gamma_1^{\mathrm{T}} \Phi_1 \Gamma_1 - \Gamma_2^{\mathrm{T}} \Phi_2 \Gamma_2, E_3 = e_{zi\mu}^{\mathrm{T}} e_{zi\mu} - e_{14}^{\mathrm{T}} \gamma^2 I e_{14},$$
$$E_{12} = \left[\sqrt{1-\alpha_1}\Lambda_3^{\mathrm{T}} Y_1 \sqrt{\alpha_1}\Lambda_4^{\mathrm{T}} Y_1^{\mathrm{T}} \sqrt{1-\alpha_2}\Lambda_5^{\mathrm{T}} Y_2 \sqrt{\alpha_2}\Lambda_4^{\mathrm{T}} Y_2^{\mathrm{T}}\right],$$
$$\Phi_1 = \begin{bmatrix} (2-\alpha_1)\bar{S}_1 + (1-\alpha_1)\varepsilon_1 I & Y_1 \\ & (1+\alpha_1)\bar{S}_1 + \alpha_1 \varepsilon_1 I \end{bmatrix},$$
$$\Phi_2 = \begin{bmatrix} (2-\alpha_2)\bar{S}_2 + (1-\alpha_2)\varepsilon_2 I & Y_2 \\ & (1+\alpha_2)\bar{S}_2 + \alpha_2 \varepsilon_2 I \end{bmatrix},$$

$$\Theta_1 = \sum_{j=1}^{N} \pi_{ij} P_j + P_i\left(2A_i + 2B_i K + \lambda_{i1}^{-1} N_i^{\mathrm{T}} N_i + \lambda_{i3}^{-1} N_{ki}^{\mathrm{T}} N_{ki} + \lambda_{i1} M_i M_i^{\mathrm{T}} P_i^{\mathrm{T}}\right.$$
$$\left. + \lambda_{i3} B_i M_i M_i^{\mathrm{T}} P_i^{\mathrm{T}} B_i^{\mathrm{T}} + \lambda_{i6} B_{di} B_{di}^{\mathrm{T}} P_i^{\mathrm{T}}\right) + \lambda_{i5}^{-1} T^2$$

$$\Theta_2 = 2A_{di} + 2B_{di} K_{di} + \lambda_{i2}^{-1} N_{di}^{\mathrm{T}} N_{di} + \lambda_{i4}^{-1} N_{kdi}^{\mathrm{T}} N_{kdi} + \lambda_{i2} M_i M_i^{\mathrm{T}} P_i^{\mathrm{T}} + \lambda_{i4} B_i M_i M_i^{\mathrm{T}} P_i^{\mathrm{T}} B_i^{\mathrm{T}},$$
$$E_{22} = diag\left\{-\bar{S}_1, -\bar{S}_1, -\bar{S}_2, -\bar{S}_2\right\},$$

$$\Lambda_1 = \left[\left(d(t) - d_1\right) e_6^{\mathrm{T}} \ \left(d_2 - d(t)\right) e_7^{\mathrm{T}} \ d(t) e_8^{\mathrm{T}} \ \left(d(t) - d_1\right)^2 e_{10}^{\mathrm{T}} \ \left(d_2 - d(t)\right)^2 e_{11}^{\mathrm{T}} \ d(t)^2 e_{12}^{\mathrm{T}}\right]^{\mathrm{T}},$$
$$\Gamma_1 = \left[\Lambda_3^{\mathrm{T}} \ \Lambda_4^{\mathrm{T}}\right]^{\mathrm{T}}, \Gamma_2 = \left[\Lambda_5^{\mathrm{T}} \ \Lambda_4^{\mathrm{T}}\right]^{\mathrm{T}},$$
$$\Lambda_2 = \left[E^{\mathrm{T}} e_2^{\mathrm{T}} - mt E^{\mathrm{T}} e_4^{\mathrm{T}} \ \ mt E^{\mathrm{T}} e_4^{\mathrm{T}} - E^{\mathrm{T}} e_3^{\mathrm{T}} \ \ e_1^{\mathrm{T}} - mt E^{\mathrm{T}} e_4^{\mathrm{T}} \ \ \left(d(t) - d_1\right) E^{\mathrm{T}} e_1^{\mathrm{T}} - \left(d(t) - d_1\right) e_6^{\mathrm{T}}\right.$$
$$\left.\left(d_2 - d(t)\right) E^{\mathrm{T}} e_1^{\mathrm{T}} - \left(d_2 - d(t)\right) e_7^{\mathrm{T}} \ \ d(t) E^{\mathrm{T}} e_1^{\mathrm{T}} - d(t) e_8^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$\Lambda_3 = \left[r_1^{\mathrm{T}} \ r_2^{\mathrm{T}} \ r_3^{\mathrm{T}}\right]^{\mathrm{T}}, \Lambda_4 = \left[r_4^{\mathrm{T}} \ r_5^{\mathrm{T}} \ r_6^{\mathrm{T}}\right]^{\mathrm{T}}, \Lambda_5 = \left[r_7^{\mathrm{T}} \ r_8^{\mathrm{T}} \ r_9^{\mathrm{T}}\right]^{\mathrm{T}}, \Lambda_6 = \left[r_{10}, r_{11}\right]^{\mathrm{T}}, \Lambda_7 = \left[r_{12}, r_{13}\right]^{\mathrm{T}},$$
$$r_1 = e_1 - e_4, r_2 = e_1 + e_4 - 2e_8, r_3 = e_1 - e_4 - 6e_8 + 12e_{12},$$
$$r_4 = e_4 - e_3, r_5 = e_3 + e_4 - 2e_7, r_6 = e_3 - e_4 - 6e_7 + 12e_{11},$$
$$r_7 = e_2 - e_4, r_8 = e_2 + e_4 - 2e_6, r_9 = e_2 - e_4 - 6e_6 + 12e_{10},$$
$$r_{10} = e_1 - e_9, r_{11} = e_1 + 2e_9 - 6e_{13}, r_{12} = e_2 - e_9, r_{13} = e_2 - 4e_9 + 6e_{13},$$

$$\mu_1 = (1-\alpha_1)\Lambda_3^{\mathrm{T}} Y_1 \bar{S}_1^{-1} Y_1 \Lambda_3 + \alpha_1 \Lambda_4^{\mathrm{T}} Y_1 \bar{S}_1^{-1} Y_1 \Lambda_4, \mu_2 = (1-\alpha_2)\Lambda_5^{\mathrm{T}} Y_2 \bar{S}_2^{-1} Y_2 \Lambda_5 + \alpha_2 \Lambda_6^{\mathrm{T}} Y_2 \bar{S}_2^{-1} Y_2 \Lambda_6,$$

$$\alpha_1 = \frac{d(t)}{d_2}, \alpha_2 = \frac{d(t) - d_1}{d_{12}}, mt = 1 - \dot{d}(t), \bar{S}_1 = diag\left\{S_1, 3S_1, 5S_1\right\}, \bar{S}_{2i} = diag\left\{S_2, 3S_2, 5S_2\right\},$$
$$\bar{Z}_1 = diag\left\{2Z_1, 4Z_1\right\}, \bar{Z}_2 = diag\left\{2Z_2, 4Z_2\right\},$$
$$e_i = \left[0_{(2n+k)\times(i-1)(2n+k)} \ I_{(2n+k)} \ 0_{(2n+k)\times(13-i)(2n+k)} \ 0_{(2n+k)\times(m+q+l)}\right], i = 1, \cdots, 14.$$

证明：首先证明系统是正则的、无脉冲的，存在 M，N 满足

$$MEN=\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, MA_iN=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, M^{-T}P_iN=\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix},$$

且 $E^{T}P_i=P_i^{T}E$，可知 $P_{12}=0$。将左右乘以 N 和 N^{T} 有 $\Omega_{i11}<0$，则 $A_{22}^{T}P_{22}+P_{22}^{T}A_{22}<0$，故 A_{22} 是非奇异的。因此 $\det(A_{22})\neq 0$ 和 $\det(\det(sE-A_i))=r=rank(E)$，则 (E,A_i) 是正则的、无脉冲的。同样地，进行左右分别乘以 $[I \ \cdots \ I]_{7\times1}^{T}$ 和 $[I \ \cdots \ I]_{7\times1}$ 有 $(A_i+A_{di})^{T}P_i+P_i^{T}(A_i+A_{di})+\sum_{j=1}^{s}\pi_{ij}E^{T}P_jE<0$ 可知 (E,A_i+A_{di}) 是正则，无脉冲的。根据[2]中定义 1 系统是正则，无脉冲的。接下来，证明系统是渐进稳定，构造 LKF 函数如下。

$$V(x_t)=\sum_{k=1}^{5}V_k,$$

$$V_1(x_t)=x^{T}(t)E^{T}P_ix(t)+\eta^{T}(t)P_i\eta(t),$$

$$V_2(x_t)=\int_{t-d_1}^{t}x^{T}(s)R_{1i}x(s)\mathrm{d}s+\int_{-d_1}^{0}\int_{t-d_1}^{t}x^{T}(s)Q_1x(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_3(x_t)=\int_{t-d_2}^{t-d(t)}x^{T}(s)R_{2i}x(s)\mathrm{d}s+\int_{t-d(t)}^{t}x^{T}(s)R_{3i}x(s)\mathrm{d}s+\int_{-d_2}^{0}\int_{t+\theta}^{t}x^{T}(s)Q_2x(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_4(x_t)=d_2\int_{-d_2}^{0}\int_{t+\theta}^{t}\dot{x}^{T}(s)E^{T}S_1E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}\dot{x}^{T}(s)E^{T}S_2E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_5(x_t)=\int_{t-d_1}^{t}\int_{u}^{t}\int_{\theta}^{t}\dot{x}^{T}(s)E^{T}Z_1E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}u+\int_{t-d_1}^{t}\int_{t-d_1}^{u}\int_{\theta}^{t}\dot{x}^{T}(s)E^{T}Z_2E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\mathrm{d}u,$$

$$\eta(t)=\Big[\int_{t-d(t)}^{t-d_1}x^{T}(t)E^{T}\mathrm{d}s \ \int_{t-d_2}^{t-d(t)}x^{T}(t)E^{T}\mathrm{d}s \ \int_{t-d(t)}^{t}x^{T}(t)E^{T}\mathrm{d}s \ \int_{t-d(t)}^{t-d_1}\int_{\theta}^{t}x^{T}(t)E^{T}\mathrm{d}s\mathrm{d}\theta$$
$$\int_{t-d_2}^{t-d(t)}\int_{\theta}^{t}x^{T}(t)E^{T}\mathrm{d}s\mathrm{d}\theta \ \int_{t-d(t)}^{t}\int_{\theta}^{t}x^{T}(t)E^{T}\mathrm{d}s\mathrm{d}\theta\Big]^{T}.$$

L 为弱无穷小算子，有：

$$LV_1(x_t)=2\Psi^{T}(t)\Lambda_1^{T}P_i\Lambda_2\Psi(t)+2x^{T}(t)EP_i\dot{x}(t)+\Psi^{T}(t)\Lambda_1^{T}\sum_{j=1}^{N}\pi_{ij}P_j\Lambda_1\Psi(t)+x^{T}(t)E^{T}\sum_{j=1}^{N}\pi_{ij}P_jx(t)$$
$$=\Psi^{T}(t)\left[sym(\Lambda_1^{T}P_i\Lambda_2)+\Lambda_1^{T}\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)\Lambda_1\right]\Psi(t)+x^{T}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t)+sym\left[x^{T}(t)E^{T}P_i\dot{x}(t)\right]$$
$$=\Psi^{T}(t)\left[sym(\Lambda_1^{T}P_i\Lambda_2)+\Lambda_1^{T}\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)\Lambda_1\right]\Psi(t)+x^{T}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t)+2x^{T}(t)P_iA_ix(t)$$
$$+2x^{T}(t)P_i\Delta A_ix(t)+2x^{T}(t)P_iB_i\Delta K_ix(t)+2x^{T}(t)P_iA_{di}x(t-d(t))+2x^{T}(t)P_i\Delta A_{di}x(t-d(t))$$
$$+2x^{T}(t)P_iB_iK_ix(t)+2x^{T}(t)P_iB_{di}K_{di}x(t-d(t))+2x^{T}(t)P_iB_{di}\Delta K_{di}x(t-d(t))$$
$$+2x^{T}(t)P_iB_if(t)+2x^{T}(t)P_iB_{di}f(t-d(t)),$$

$$LV_2(x_t)=\Psi^{T}(t)\left(e_1^{T}(R_{1i}+d_1Q_1)e_1-e_2^{T}R_{1i}e_2\right)\Psi(t)+\int_{t-d_1}^{t}x^{T}(s)\left(\sum_{j=1}^{N}\pi_{ij}R_{1j}-Q_1\right)x(s)\mathrm{d}s,$$

$$LV_3(x_t)=\Psi^{T}(t)\left(e_1^{T}(R_{3i}+d_2Q_2)e_1+mte_4^{T}(R_{2i}-R_{3i})e_4-e_3^{T}R_{2i}e_3\right)\Psi(t)$$
$$+\int_{t-d_2}^{t-d(t)}x^{T}(s)\left(\sum_{j=1}^{N}\pi_{ij}(R_{2j}-R_{3j})\right)x^{T}(s)\mathrm{d}s+\int_{t-d_2}^{t}x^{T}(s)\left(\sum_{j=1}^{N}\pi_{ij}R_{3j}-Q_2\right)x(s)\mathrm{d}s,$$

$$LV_4(x_t)=\Psi^{\mathrm{T}}(t)\left(e_5^{\mathrm{T}}\left(d_2^2S_1+d_{12}^2S_2\right)e_5\right)\Psi(t)-d_2\int_{t-d_2}^{t}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)S_1E\dot{x}(s)\mathrm{d}s$$
$$-d_{12}\int_{t-d_2}^{t-d_1}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)S_2E\dot{x}(s)\mathrm{d}s,$$

$$LV_5(x_t)=\Psi^{\mathrm{T}}(t)\left(\frac{d_1^2}{2}e_5^{\mathrm{T}}(Z_1+Z_2)e_5\right)\Psi(t)-\int_{t-d_1}^{t}\int_{\theta}^{t}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)Z_1E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$
$$-\int_{t-d_1}^{t}\int_{t-d_1}^{\theta}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)Z_2E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$\Psi(t)=\left[\psi_1^{\mathrm{T}}(t)\ \ \psi_2^{\mathrm{T}}(t)\ \ \psi_3^{\mathrm{T}}(t)\ \ w^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$

$$\psi_1(t)=\left[x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t-d_1)\ \ x^{\mathrm{T}}(t-d_2)\ \ x^{\mathrm{T}}(t-d(t))\ \ \dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$\psi_2(t)=\left[\frac{1}{d(t)-d_1}\int_{t-d(t)}^{t-d_1}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\ \ \frac{1}{d_2-d(t)}\int_{t-d_2}^{t-d(t)}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\ \ \frac{1}{d(t)}\int_{t-d(t)}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\ \ \frac{1}{d_1}\int_{t-d_1}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\right]^{\mathrm{T}},$$

$$\psi_3(t)=\left[\frac{1}{(d(t)-d_1)^2}\int_{t-d(t)}^{t-d_1}\int_{\theta}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\mathrm{d}\theta\ \ \frac{1}{(d_2-d(t))^2}\int_{t-d_2}^{t-d(t)}\int_{\theta}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\mathrm{d}\theta\right.$$
$$\left.\frac{1}{(d(t))^2}\int_{t-d(t)}^{t}\int_{\theta}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\mathrm{d}\theta\ \ \frac{1}{d_1^2}\int_{t-d_1}^{t}\int_{\theta}^{t}x^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s\right]^{\mathrm{T}}.$$

由[8]中引理 2，4 和假设 1，2

$$2x^{\mathrm{T}}(t)P_i\Delta A_ix(t)=2x^{\mathrm{T}}(t)P_iM_iF_i(t)N_ix(t)\le\lambda_{i1}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}x(t)+\lambda_{i1}^{-1}x^{\mathrm{T}}(t)N_i^{\mathrm{T}}N_ix(t),$$

$$2x^{\mathrm{T}}(t)P_i\Delta A_{di}x(t-d(t))=2x^{\mathrm{T}}(t)PM_iF_i(t)N_{di}x(t-d(t))$$
$$\le\lambda_{i2}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}x_1(t-d(t))+\lambda_{i2}^{-1}x^{\mathrm{T}}(t)N_{di}^{\mathrm{T}}N_{di}x(t-d(t)),$$

$$2x^{\mathrm{T}}(t)P_iB_i\Delta K_ix(t)=2x^{\mathrm{T}}(t)P_iB_iM_iF_i(t)N_{ki}x(t)\le\lambda_{i3}x^{\mathrm{T}}(t)P_iB_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}}x(t)+\lambda_{i3}^{-1}x^{\mathrm{T}}(t)N_{ki}^{\mathrm{T}}N_{ki}x(t),$$

$$2x^{\mathrm{T}}(t)P_iB_i\Delta K_{kdi}x(t-d(t))\le\lambda_{i4}x^{\mathrm{T}}(t)P_iB_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}}x(t-d(t))+\lambda_{i4}^{-1}x^{\mathrm{T}}(t)N_{kdi}^{\mathrm{T}}N_{kdi}x(t-d(t)),$$

$$2x^{\mathrm{T}}(t)P_iB_if(t)\le\lambda_{i5}x^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_i^{\mathrm{T}}x(t)+\lambda_{i5}^{-1}f^{\mathrm{T}}(t)f(t)\le\lambda_{i5}x^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_i^{\mathrm{T}}x(t)+\lambda_{i5}^{-1}x^{\mathrm{T}}(t)T^2x(t),$$

$$2x^{\mathrm{T}}(t)P_iB_{di}f(t-d(t))\le\lambda_{i6}x^{\mathrm{T}}(t)P_iB_{di}B_{di}^{\mathrm{T}}P_i^{\mathrm{T}}x(t)+\lambda_{i6}^{-1}f^{\mathrm{T}}(t-d(t))f(t-d(t))$$
$$\le\lambda_{i6}x^{\mathrm{T}}(t)P_iB_{di}B_{di}^{T}P_i^{T}x(t)+\lambda_{i6}^{-1}x^{\mathrm{T}}(t-d(t))T^2x(t-d(t)).$$

$$LV_1(x_t)\le\Psi^{\mathrm{T}}(t)\left[sym\left(\Lambda_1^{\mathrm{T}}P_i\Lambda_2\right)+\Lambda_1^{\mathrm{T}}\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)\Lambda_1\right]\Psi(t)+x^{\mathrm{T}}(t)\left[\sum_{j=1}^{N}\pi_{ij}P_j+P_i\left(2A_i+2B_iK+\lambda_{i1}^{-1}N_i^{\mathrm{T}}N_i\right.\right.$$
$$\left.\left.+\lambda_{i3}^{-1}N_{ki}^{\mathrm{T}}N_{ki}+\lambda_{i1}M_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}+\lambda_{i3}B_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}}+\lambda_{i6}B_{di}B_{di}^{\mathrm{T}}P_i^{\mathrm{T}}\right)+\lambda_{i5}^{-1}T^2\right]x(t)$$
$$+x^{\mathrm{T}}(t)P_i\left(2A_{di}+2B_{di}K_{di}+\lambda_{i2}^{-1}N_{di}^{\mathrm{T}}N_{di}+\lambda_{i4}^{-1}N_{kdi}^{\mathrm{T}}N_{kdi}+\lambda_{i2}M_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}\right.$$
$$\left.+\lambda_{i4}B_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}}\right)x(t-d(t))+\lambda_{i6}^{-1}x^{\mathrm{T}}(t-d(t))T^2x(t-d(t))$$
$$=\Psi^{\mathrm{T}}(t)\left[sym\left(\Lambda_1^{\mathrm{T}}P_i\Lambda_2\right)+\Lambda_1^{\mathrm{T}}\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)\Lambda_1+e_1^{\mathrm{T}}\Theta_1e_1+e_1^{\mathrm{T}}P_i\Theta_2e_2+e_2^{\mathrm{T}}\lambda_{i6}^{-1}T^2e_2\right]\Psi(t),$$

$$\Theta_1=\sum_{j=1}^{N}\pi_{ij}P_j+P_i\left(2A_i+2B_iK+\lambda_{i1}^{-1}N_i^{\mathrm{T}}N_i+\lambda_{i3}^{-1}N_{ki}^{\mathrm{T}}N_{ki}+\lambda_{i1}M_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}+\lambda_{i3}B_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}}+\lambda_{i6}B_{di}B_{di}^{\mathrm{T}}P_i^{\mathrm{T}}\right)+\lambda_{i5}^{-1}T^2,$$

$$\Theta_2=2A_{di}+2B_{di}K_{di}+\lambda_{i2}^{-1}N_{di}^{T}N_{di}+\lambda_{i4}^{-1}N_{kdi}^{\mathrm{T}}N_{kdi}+\lambda_{i2}M_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}+\lambda_{i4}B_iM_iM_i^{\mathrm{T}}P_i^{\mathrm{T}}B_i^{\mathrm{T}},$$

[9]结论 1 和[10]引理 3，有

$$\begin{aligned}&-d_2\int_{t-d_2}^{t}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)S_1E\dot{x}(s)\mathrm{d}s=-d_2\int_{t-d(t)}^{t}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)S_1E\dot{x}(s)\mathrm{d}s-d_2\int_{t-d_2}^{t-d(t)}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)S_2E\dot{x}(s)\mathrm{d}s\\
&\le\Psi^{\mathrm{T}}(t)\left[-\frac{1}{\alpha_1}\left(r_1^{\mathrm{T}}S_1r_1+3r_2^{\mathrm{T}}S_1r_2+5r_3^{\mathrm{T}}S_1r_3\right)-\frac{1}{1-\alpha_1}\left(r_4^{\mathrm{T}}S_1r_4+3r_5^{\mathrm{T}}S_1r_5+5r_6^{\mathrm{T}}S_1r_6\right)\right]\Psi(t)\\
&\le\Psi^{\mathrm{T}}(t)\Gamma_1^{\mathrm{T}}\begin{bmatrix}\bar{S}_1+(1-\alpha_1)\left(\bar{S}_1+\varepsilon_1I-Y_1\bar{S}_1^{-1}Y_1^{\mathrm{T}}\right) & Y_1\\ & \bar{S}_1+\alpha_1\left(\bar{S}_1+\varepsilon_1I-Y_1^{\mathrm{T}}\bar{S}_1^{-1}Y_1\right)\end{bmatrix}\Gamma_1\Psi(t)\\
&=\Psi^{\mathrm{T}}(t)\left(\mu_1-\Gamma_1^{\mathrm{T}}\Phi_1\Gamma_1\right)\Psi^{\mathrm{T}}(t)\end{aligned}$$

$$-d_{12}\int_{t-d_2}^{t-d_1}E\dot{x}^{\mathrm{T}}(s)S_2E\dot{x}(s)\le\Psi^{\mathrm{T}}(t)\left(\mu_2-\Gamma_2^{\mathrm{T}}\Phi_2\Gamma_2\right)\Psi^{\mathrm{T}}(t)$$

$$\begin{aligned}&\mu_1=(1-\alpha_1)\Lambda_3^{\mathrm{T}}Y_1\bar{S}_1^{-1}Y_1\Lambda_3+\alpha_1\Lambda_4^{\mathrm{T}}Y_1\bar{S}_1^{-1}Y_1\Lambda_4,\\
&\mu_2=(1-\alpha_2)\Lambda_5^{\mathrm{T}}Y_2\bar{S}_2^{-1}Y_2\Lambda_5+\alpha_2\Lambda_6^{\mathrm{T}}Y_2\bar{S}_2^{-1}Y_2\Lambda_6,\bar{S}_n=diag\left\{S_n,3S_n,5S_n\right\},\end{aligned}$$

根据[11]中(9)，(10)，我们得到

$$\begin{aligned}&-\int_{t-d_1}^{t}\int_{\theta}^{t}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)Z_1E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\le-\Psi^{\mathrm{T}}(t)\left[2r_{10}^{\mathrm{T}}Z_1r_{10}+4r_{11}^{\mathrm{T}}Z_1r_{11}\right]\Psi(t)=\Psi^{\mathrm{T}}(t)\left[-\left(\Lambda_6^{\mathrm{T}}\bar{Z}_1\Lambda_6\right)\right]\Psi(t)\\
&-\int_{t-d_1}^{t}\int_{t-d_1}^{\theta}E^{\mathrm{T}}\dot{x}^{\mathrm{T}}(s)Z_2E\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\le-\Psi^{\mathrm{T}}(t)\left[2r_{12}^{\mathrm{T}}Z_2r_{12}+4r_{13}^{\mathrm{T}}Z_2r_{13}\right]\Psi(t)=\Psi^{\mathrm{T}}(t)\left[-\left(\Lambda_7^{\mathrm{T}}\bar{Z}_2\Lambda_7\right)\right]\Psi(t).\end{aligned}$$

通过上述，我们可以得到

$$LV(x_t)\le\xi^{\mathrm{T}}(t)\Omega_0\xi(t),$$

其中 $\Omega_0=E_{1i\mu}\big|_{d(t),\dot{d}(t)}+E_2+\eta_1+\eta_2$，由 Schur 补，我们有，

$$\Omega_0=E_{1i\mu}\big|_{d(t),\dot{d}(t)}+E_2+\eta_1+\eta_2<0,$$

$$LV(x_t)<0,$$

这意味着 $LV(x_t)\le-\varsigma x^{\mathrm{T}}(t)x(t)$，使用 Dynkin 公式，所有 $i\in I$，$T>0$，遵循 $i\in I$，$T>0$，

$$E\left\{V\left(x(T),r(T)\right)\right\}-V\left(x(0),r(0)\right)\le-\varsigma E\left\{\int_0^T x^{\mathrm{T}}(s)x(s)\middle|\left(x(0),r(0)\right)\right\},$$

此外，让 $T\to\infty$，$E\left\{\int_0^T x^{\mathrm{T}}(s)x(s)\middle|\left(x(0),r(0)\right)\right\}\le\frac{1}{\varsigma}V\left(x(0),r(0)\right)<\infty$。

根据定义 2.1，系统(2.2)是随机稳定的。接下来我们考虑 H_∞性能函数 J，

$$\begin{aligned}J&=\int_0^\infty z^{\mathrm{T}}(t)z(t)-\gamma^2w^{\mathrm{T}}(t)w(t)\mathrm{d}t\le\int_0^\infty LV(x_t)+z^{\mathrm{T}}(t)z(t)-\gamma^2w^{\mathrm{T}}(t)w(t)\mathrm{d}t\\
&=\int_0^\infty\Psi^{\mathrm{T}}(t)\Omega\Psi(t)\mathrm{d}t,\end{aligned}$$

我们有 $J<0$。H_∞性能已验证。根据 GUTR 矩阵的定义，我们要探讨以下三种情况下的上述不等式。

情况 I $i\notin U_k^i$，$U_k^i=\left\{k_1^i,\cdots,k_{m_i}^i\right\}$，存在正定矩阵，$V_{ij}\in R^{n\times n}\left(i\notin U_k^i,j\in U_k^i\right)$及 $l\in U_{uk}^i$ 有

$$\begin{bmatrix} E^{[m]} & E_{12} & e_{zi\mu}^{\mathrm{T}} & \Pi_1^{\mathrm{T}} E\left(P_{k_1^i} - F_i\right) & \cdots & \Pi_1^{\mathrm{T}} E\left(P_{ik_{mi}^i} - F_i\right) \\ * & E_{22} & 0 & 0 & \cdots & 0 \\ * & * & -I & 0 & \cdots & 0 \\ * & * & * & -V_{ik_1^i} & \cdots & 0 \\ * & * & * & * & \ddots & \vdots \\ * & * & * & * & * & -V_{ik_{mi}^i} \end{bmatrix} < 0,$$

证明 $i \notin U_k^i$，应该注意的是，在这种情况下 $\sum_{j \in U_k^i, j \neq i} \pi_{ij} = -\pi_{ii} - \sum_{j \in U_k^i} \pi_{ij}$，$\pi_{ij} \geq 0$，且必须有 $l \in U_k^i$，$l \neq j$，使 $E^{\mathrm{T}} \bar{P}_l E - E^{\mathrm{T}} \bar{P}_j E \geq 0$，定义

$$\begin{aligned} \Omega = \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \left(\sum_{j \in U_k^i} \pi_{ij} P_j + \pi_{ii} P_i + \sum_{j \in U_k^i, j \neq i} \pi_{ij} P_j \right) \Lambda_1 \\ + x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(\sum_{j \in U_k^i} \pi_{ij} P_j + \pi_{ii} P_i + \sum_{j \in U_k^i, j \neq i} \pi_{ij} P_j \right) x(t), \end{aligned} \tag{2.6}$$

$$\begin{aligned} \Omega &= \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \sum_{j \in U_k^i} \pi_{ij} P_j \Lambda_1 + \Lambda_1^{\mathrm{T}} \pi_{ii} P_i \Lambda_1 + \Lambda_1^{\mathrm{T}} \sum_{j \in U_k^i, j \neq i} \pi_{ij} P_j \Lambda_1 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j \in U_k^i} \pi_{ij} P_j x(t) \\ &\quad + x^{\mathrm{T}}(t) E^{\mathrm{T}} \pi_{ii} P_i x(t) + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j \in U_k^i, j \neq i} \pi_{ij} P_j x(t) \\ &\leq \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \sum_{j \in U_k^i} \pi_{ij} P_j \Lambda_1 + \Lambda_1^{\mathrm{T}} \pi_{ii} P_i \Lambda_1 + \Lambda_1^{\mathrm{T}} \left(-\pi_{ii} - \sum_{j \in U_k^i} \pi_{ij} \right) P_l \Lambda_1 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j \in U_k^i} \pi_{ij} P_j x(t) \\ &\quad + x^{\mathrm{T}}(t) E^{\mathrm{T}} \pi_{ii} P_i x(t) + x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(-\pi_{ii} - \sum_{j \in U_k^i} \pi_{ij} \right) P_l x(t) \\ &= \Xi + E_2 + E_3 + \sum_{j \in U_k^i} \pi_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \sum_{j \in U_k^i} \pi_{ij} x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(P_j - P_l \right) x(t) \\ &= \Xi + E_2 + E_3 + \sum_{j \in U_k^i} \left(\hat{\pi}_{ij} + \pi_{ij} \right) \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \sum_{j \in U_k^i} \left(\hat{\pi}_{ij} + \pi_{ij} \right) x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(P_j - P_l \right) x(t) \\ &= \Xi + E_2 + E_3 + \sum_{j \in U_k^i} \hat{\pi}_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \sum_{j \in U_k^i} \pi_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \sum_{j \in U_k^i} \hat{\pi}_{ij} x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(P_j - P_l \right) x(t) \\ &\quad + \sum_{j \in U_k^i} \pi_{ij} x^{\mathrm{T}}(t) E^{\mathrm{T}} \left(P_j - P_l \right) x(t). \end{aligned} \tag{2.7}$$

此外，由于引理 2.1，可以得到

$$\begin{aligned} &\sum_{j \in U_k^i} \pi_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \sum_{j \in U_k^i} \pi_{ij} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l \right) x(t) \\ &= \sum_{j \in U_k^i} \left[\frac{1}{2} \pi_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 + \frac{1}{2} \pi_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 \right] \\ &\quad + \sum_{j \in U_k^i} \left[\frac{1}{2} \pi_{ij} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l \right) x(t) + \frac{1}{2} \pi_{ij} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l \right) x(t) \right] \\ &\leq \sum_{j \in U_k^i} \left[\frac{\delta_{ij1}^2}{4} V_{ij1} + \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 V_{ij1}^{-1} \Lambda_1^{\mathrm{T}} \left(P_j - P_l \right) \Lambda_1 \right] \\ &\quad + \sum_{j \in U_k^i} \left[\frac{\delta_{ij2}^2}{4} V_{ij2} + E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l \right) x(t) V_{ij2}^{-1} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l \right) x(t) \right], \end{aligned} \tag{2.8}$$

从(2.6)~(2.8)中，我们有

$$\begin{aligned}\Omega \le{}& \Xi + E_2 + \sum_{j\in U_k^i}\hat{\pi}_{ij}\Lambda_1^{\mathrm{T}}\left(P_j - P_l\right)\Lambda_1 + \sum_{j\in U_k^i}\frac{\delta_{ij1}^2}{4}V_{ij1} + \sum_{j\in U_k^i}\left[\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1 V_{ij1}^{-1}\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1\right]\\ &+\sum_{j\in U_k^i}\frac{\delta_{ij2}^2}{4}V_{ij2} + \sum_{j\in U_k^i}\left[E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t)V_{ij2}^{-1}E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t)\right]\\ &+\sum_{j\in U_k^i}\hat{\pi}_{ij}E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t)\end{aligned}\tag{2.9}$$

如果，

$$\begin{aligned}&\Xi + E_2 + \sum_{j\in U_k^i}\hat{\pi}_{ij}\Lambda_1^{\mathrm{T}}\left(P_j - P_l\right)\Lambda_1 + \sum_{j\in U_k^i}\frac{\delta_{ij1}^2}{4}V_{ij1} + \sum_{j\in U_k^i}\left[\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1 V_{ij1}^{-1}\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1\right] + \sum_{j\in U_k^i}\frac{\delta_{ij2}^2}{4}V_{ij2}\\ &+\sum_{j\in U_k^i}\left[E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t)V_{ij2}^{-1}E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t)\right] + \sum_{j\in U_k^i}\hat{\pi}_{ij}E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(P_j-P_l\right)x(t) < 0\end{aligned}\tag{2.10}$$

则 $\Omega<0$，由引理 2.2 有(2.10)成立。可以看出

$$LV\left(x_t\right) \le \gamma^2 w^{\mathrm{T}}(t)w(t) - z^{\mathrm{T}}(t)z(t).$$

由定义 2.2，系统(2.2)在 H_∞扰动水平为零时，是随机允许的 γ。

情况 II $i\in U_k^i$，$U_k^i \ne \varnothing$，存在正定矩阵，$G_{ij}\in R^{n\times n}\left(i,j\in U_k^i, l\in U_{uk}^i\right)$有

$$\begin{bmatrix} E^{[m]} & E_{12} & e_{zi\mu}^{\mathrm{T}} & \Pi_1^{\mathrm{T}}E\left(P_{k_1^i}-F_i\right) & \cdots & \Pi_1^{\mathrm{T}}E\left(P_{ik_{mi}^i}-F_i\right)\\ * & E_{22} & 0 & 0 & \cdots & 0\\ * & * & -I & 0 & \cdots & 0\\ * & * & * & -G_{ik_1^i} & \cdots & 0\\ * & * & * & * & \ddots & \vdots\\ * & * & * & * & * & -G_{ik_{mi}^i}\end{bmatrix} < 0,$$

证明 $i\in U_k^i$ 且 $U_{uk}^i \ne\varnothing$，必须有 $l\in U_{uk}^i$，$E^{\mathrm{T}}P(l)E - E^{\mathrm{T}}P(j)E \ge 0$，$j\in U_{uk}^i$。因为它认为

$$\Omega = \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}}\left(\sum_{j\in U_k^i}\pi_{ij}P_j + \pi_{ii}P_i + \sum_{j\in U_{uk}^i}\pi_{ij}P_j\right)\Lambda_1 + x^{\mathrm{T}}(t)E^{\mathrm{T}}\left(\sum_{j\in U_k^i}\pi_{ij}P_j + \pi_{ii}P_i + \sum_{j\in U_{uk}^i}\pi_{ij}P_j\right)x(t),$$

然后我们有

$$\begin{aligned}\Omega \le{}& \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}}\left(\sum_{j\in U_k^i}\pi_{ij}P_j + \sum_{j\in U_{uk}^i}\pi_{ij}P_l\right)\Lambda_1 + E^{\mathrm{T}}x^{\mathrm{T}}(t)\left(\sum_{j\in U_k^i}\pi_{ij}P_j + \sum_{j\in U_{uk}^i}\pi_{ij}P_l\right)x(t)\\ ={}& \Xi + E_2 + E_3 + \sum_{j\in U_k^i}\pi_{ij}\Lambda_1^{\mathrm{T}}P_j\Lambda_1 - \sum_{j\in U_k^i}\pi_{ij}\Lambda_1^{\mathrm{T}}P_l\Lambda_1 + \sum_{j\in U_k^i}\pi_{ij}x^{\mathrm{T}}(t)E^{\mathrm{T}}P_jx(t) - \sum_{j\in U_k^i}\pi_{ij}x^{\mathrm{T}}(t)E^{\mathrm{T}}P_lx(t)\\ ={}& \Xi + E_2 + E_3 + \sum_{j\in U_k^i}\left(\hat{\pi}_{ij}+\pi_{ij}\right)\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1 + \sum_{j\in U_k^i}\left(\hat{\pi}_{ij}+\pi_{ij}\right)x^{\mathrm{T}}(t)E^{\mathrm{T}}\left(P_j-P_l\right)x(t)\\ ={}& \Xi + E_2 + E_3 + \sum_{j\in U_k^i}\hat{\pi}_{ij}\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1 + \sum_{j\in U_k^i}\pi_{ij}\Lambda_1^{\mathrm{T}}\left(P_j-P_l\right)\Lambda_1 + \sum_{j\in U_k^i}\hat{\pi}_{ij}x^{\mathrm{T}}(t)E^{\mathrm{T}}\left(P_j-P_l\right)x(t)\\ &+\sum_{j\in U_k^i}\pi_{ij}x^{\mathrm{T}}(t)E^{\mathrm{T}}\left(P_j-P_l\right)x(t),\end{aligned}$$

此外，由于引理 2.2，我们可以得到

$$
\begin{aligned}
&\sum_{j\in U_k^i}\pi_{ij}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1+\sum_{j\in U_k^i}\pi_{ij}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)\\
&=\sum_{j\in U_k^i}\left[\frac12\pi_{ij}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1+\frac12\pi_{ij}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1\right]\\
&\quad+\sum_{j\in U_k^i}\left[\frac12\pi_{ij}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)+\frac12\pi_{ij}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)\right]\\
&\le\sum_{j\in U_k^i}\left[\frac{\delta_{ij1}^2}{4}G_{ij1}+\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1G_{ij1}^{-1}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1\right]\\
&\quad+\sum_{j\in U_k^i}\left[\frac{\delta_{ij2}^2}{4}G_{ij2}+E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)G_{ij2}^{-1}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)\right],
\end{aligned}
\tag{2.11}
$$

另外，我们有

$$
\begin{aligned}
\Omega\le{}&\Xi+E_2+E_3+\sum_{j\in U_k^i}\hat\pi_{ij}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1+\sum_{j\in U_k^i}\frac{\delta_{ij1}^2}{4}G_{ij1}\\
&+\sum_{j\in U_k^i}\left[\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1G_{ij1}^{-1}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1\right]\\
&+\sum_{j\in U_k^i}\hat\pi_{ij}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)+\sum_{j\in U_k^i}\frac{\delta_{ij2}^2}{4}G_{ij2}\\
&+\sum_{j\in U_k^i}\left[E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)G_{ij2}^{-1}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)\right],
\end{aligned}
\tag{2.12}
$$

如果

$$
\begin{aligned}
&\Xi+E_2+E_3+\sum_{j\in U_k^i}\hat\pi_{ij}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1+\sum_{j\in U_k^i}\left[\Lambda_1^{\mathrm T}\left(P_j-P_l\right)E^{\mathrm T}\Lambda_1G_{ij1}^{-1}\Lambda_1^{\mathrm T}\left(P_j-P_l\right)\Lambda_1\right]\\
&+\sum_{j\in U_k^i}\frac{\delta_{ij1}^2}{4}G_{ij1}+\sum_{j\in U_k^i}\left[E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)G_{ij2}^{-1}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)\right]\\
&+\sum_{j\in U_k^i}\frac{\delta_{ij2}^2}{4}G_{ij2}+\sum_{j\in U_k^i}\hat\pi_{ij}E^{\mathrm T}x^{\mathrm T}(t)\left(P_j-P_l\right)x(t)<0,
\end{aligned}
\tag{2.13}
$$

因此 $\Omega<0$，由引理 2.2 有(2.13)成立。可以看出 $LV(x_t)\le\gamma^2w^{\mathrm T}(t)w(t)-z^{\mathrm T}(t)z(t)$。由定义 2.2，系统(2.2)在 H_∞ 扰动水平为零时是随机允许的 γ。

情况 III $i\in U_k^i$，$U_{uk}^i=\varnothing$，存在正定矩阵，$L_{ij}\in R^{n\times n}\left(i,j\in U_k^i\right)$ 有

$$
\begin{bmatrix}
E^{[m]} & E_{12} & e_{zi\mu}^{\mathrm T} & \Pi_1^{\mathrm T}E\left(P_{k_1^i}-F_i\right) & \cdots & \Pi_1^{\mathrm T}E\left(P_{ik_{mi}^i}-F_i\right)\\
* & E_{22} & 0 & 0 & \cdots & 0\\
* & * & -I & 0 & \cdots & 0\\
* & * & * & -L_{ik_1^i} & \cdots & 0\\
* & * & * & * & \ddots & \vdots\\
* & * & * & * & * & -L_{ik_{mi}^i}
\end{bmatrix}<0,
$$

证明 $i\in U_k^i$，$U_{uk}^i=\varnothing$。同样，它认为

$$\Omega = \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \pi_{ij} P_j \Lambda_1 + \Lambda_1^{\mathrm{T}} \pi_{ii} P_i \Lambda_1 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \pi_{ij} P_j x(t) + x^{\mathrm{T}}(t) E^{\mathrm{T}} \pi_{ii} P_i x(t)$$
$$= \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \pi_{ij} \left(P_j - P_i\right) \Lambda_1 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \pi_{ij} \left(P_j - P_i\right) x(t)$$
$$= \Xi + E_2 + E_3 + \Lambda_1^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \left(\hat{\pi}_{ij} + \pi_{ij}\right)\left(P_j - P_i\right) \Lambda_1 + x^{\mathrm{T}}(t) E^{\mathrm{T}} \sum_{j=1, j\neq i}^{s} \left(\hat{\pi}_{ij} + \pi_{ij}\right)\left(P_j - P_i\right) x(t)$$
$$\leq \Xi + E_2 + E_3 + \sum_{j\in U_k^i} \hat{\pi}_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_i\right) \Lambda_1 + \sum_{j\in U_k^i} \frac{\delta_{ij1}^2}{4} L_{ij1} + \sum_{j\in U_k^i} \left[\Lambda_1^{\mathrm{T}} \left(P_j - P_i\right) \Lambda_1 L_{ij1}^{-1} \Lambda_1^{\mathrm{T}} \left(P_j - P_i\right) \Lambda_1\right]$$
$$+ \sum_{j\in U_k^i} \left[E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_i\right) x(t) L_{ij2}^{-1} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_i\right) x(t)\right] + \sum_{j\in U_k^i} \hat{\pi}_{ij} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_i\right) x(t) + \sum_{j\in U_k^i} \frac{\delta_{ij2}^2}{4} L_{ij2},$$

如果

$$\Xi + E_2 + E_3 + \sum_{j\in U_k^i} \hat{\pi}_{ij} \Lambda_1^{\mathrm{T}} \left(P_j - P_l\right) \Lambda_1 + \sum_{j\in U_k^i} \frac{\delta_{ij1}^2}{4} L_{ij1} + \sum_{j\in U_k^i} \hat{\pi}_{ij} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l\right) x(t) + \sum_{j\in U_k^i} \frac{\delta_{ij2}^2}{4} L_{ij2}$$
$$+ \sum_{j\in U_k^i} \left[\Lambda_1^{\mathrm{T}} \left(P_j - P_l\right) \Lambda_1 L_{ij1}^{-1} \Lambda_1^{\mathrm{T}} \left(P_j - P_l\right) \Lambda_1\right] + \sum_{j\in U_k^i} \left[E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l\right) x(t) L_{ij2}^{-1} E^{\mathrm{T}} x^{\mathrm{T}}(t) \left(P_j - P_l\right) x(t)\right] < 0 \quad (2.14)$$

则 $\Omega < 0$，引理 2.2，(2.14)成立可以看出

$$LV(x_t) \leq \gamma^2 w^{\mathrm{T}}(t) w(t) - z^{\mathrm{T}}(t) z(t).$$

由定义 2.2 可知，系统(2.2)在扰动水平为 H_∞ 时为零时是随机允许的 γ。考虑具有恒定延迟 h_2 的 SMJs，其中的参数与[8]中的参数相同，数值示例说明本文比以前方法有效更通用见表 1：

Table 1. Upper bound contrast
表 1. 上界对比

比较结果	d
[8]	1.2362
本文 $\varepsilon = 0$	1.4731
本文 $\varepsilon = 2$	1.5836

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0.4972 & 0 \\ 0 & -0.9541 \end{bmatrix}, A_d = \begin{bmatrix} -1.4972 & 1.5415 \\ 0 & 0.5449 \end{bmatrix},$$
$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, N = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}, MEN = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

此外，我们选择

$$\Sigma = \begin{bmatrix} 0.1 & 0.4 \\ 0.2 & 0.5 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, B_d = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, B_w = \begin{bmatrix} 1 \\ 2.5 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$C_d = \begin{bmatrix} 0.3 & -0.3 \end{bmatrix}, D_w = \begin{bmatrix} 1 & 1.5 \\ 3 & 2 \end{bmatrix}, H_R = 4, \gamma = 0.1.$$

为了验证所提出的方法绘制了 $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}$ 和 $w(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}^{\mathrm{T}} \sin e^{-0.2t}$ 可以看出 SNJs 状态响应收敛于零见图 1。

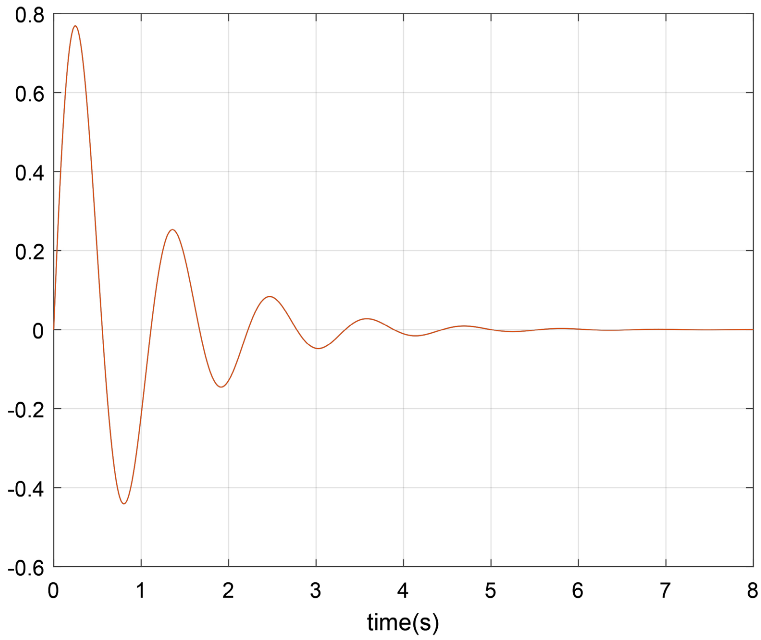

Figure 1. $x(t)$ status response with external input $w(t)$

图 1. 具有外部输入 $w(t)$ 的 $x(t)$ 状态响应

备注 2.2：与一般的 LKF 相比，乘以更多的积分项，利用了更多的时变延迟信息，在增强的 LKF 中减少保守性。(2.21)和(2.22)中的条件可以通过使用[10]引理 3 得到更紧的界限。因此，本文定理的保守性比 Wang 等[12]、Fu 和 Ma [13]更低。

3. 结论

对于给定的标量本章探讨一类非线性广义 Markov 跳跃系统的随机稳定性和 H_∞控制问题。设计新型的 Lyapunov-Krasovkii 泛函(LKF)，构造适当的 Lyapunov-Krasovskii 泛函，应用依赖参数相关的互复凸矩阵不等式(PDRCMI)、改进的 Wirtinger 不等式，Chen 提出的广义积分不等式，通过 LMI，得到保守性较低的准则。同时，引入参数依赖的互凸矩阵不等式(PDRCMI)来降低保守性，保证了非线性广义 Markov 跳跃系统渐进稳定并满足性能。此外，将自适应方法应用于 MJSs 具有现实意义，值得进一步探索。值得一提的是，我们的理论结果也可以在未来更复杂的系统。

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