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一类混沌神经网络的研究

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摘 要

本文研究了延迟混沌神经网络的同步问题，引入一种新的自适应反馈控制器(包括状态耦合控制和延迟状态耦合控制)，利用李雅普诺夫泛函研究了变系数情形下系统的同步性。最后利用数值模拟验证了结论的有效性。

关键词

自适应同步，时滞混沌神经网络，时变系数，稳定性

Study on a Class of Chaotic Neural Networks

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Abstract

This paper is concerned with the synchronization of the delayed chaotic neural networks, it has a new adaptive feedback controller which includes state coupling control

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and delayed state coupling control. We study the case where the system is a variable coefficient. Also we discuss the stability conditions of synchronization by constructing a new Lyapunov functional. Example and numerical simulation are given to illustrate the effectiveness of the results.

Keywords

Adaptive Synchronization, Delayed Chaotic Neural Networks, Time-Varying Cofficient, Stability

1. 主要模型和预备知识

同步是指不同过程在时间上的一致或相关。有很多不同的同步类型如混沌同步 [1, 2]、相位同步 [3]和集群同步 [4]。特别是由于其在许多领域的潜在应用。近几十年来，非线性混沌同步动力系统得到了广泛的研究。同步现象在许多领域得到了广泛的研究和应用。因此，对研究同步的理论和应用有很大的兴趣 [5–9]。

在同步方面，学者们分别研究了自适应控制等不同的混沌同步方案 [2]，反馈控制 [1, 2, 10, 11]，固定控制 [2, 12] 等等。 [13]考虑更复杂的系统，用类似方法研究变系数情况。在实际应用中，由于传输速度有限而交通拥堵，信息传递往往滞后。将时间延迟引入神经系统，是实际设计和应用的必要条件。在这篇文章中，我们考虑了由以下状态定义的具有时变延迟的神经网络方程：

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + I \tag{1}$$

另一种形式为

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + I \tag{2}$$

这里$x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ 代表神经元的状态向量；$C = diag(c_1, c_2, \cdots, c_n) > 0$ (一个正的对角矩阵)，$A = (a_{ij})_{n\times n}$和$B = (b_{ij})_{n\times n}$分别指的是连接权重矩阵和延迟连接矩阵；$I = (I_1, I_2, \cdots, I_n)^{\mathrm{T}} \in \mathbb{R}^n$ 是一个连续的外部输入；$\tau(t) \geq 0$ 是传输时滞; $f(x(t)) = f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))^{\mathrm{T}} \in \mathbb{R}^n$ 代表神经元的激活函数。

从系统方程如下所示：

$$\dot{y}_i(t) = -c_i(t)y_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j(y_j(t-\tau(t))) + I + U_i(t) \tag{3}$$

另一种形式为

$$\dot{y}(t) = -C(t)y(t) + A(t)f(y(t)) + B(t)f(y(t-\tau(t))) + I + U(t) \tag{4}$$

这里$U(t) = [u_1(t), u_2(t), \cdots, u_n(t)]^{\mathrm{T}}$ 是控制器。令$e(t) = y(t) + mx(t)$，并且自适应反馈控制器设计为$U(t) = L(y(t) - x(t)) + K(y(t-\tau(t)) - x(t-\tau(t)))$，这里的矩阵$L = \mathrm{diag}\,(l_1, l_2, \cdots, l_n) > 0, K = \mathrm{diag}\,(k_1, k_2, \cdots, k_n) > 0$。然后，同步的误差系统可以写成：

$$\begin{aligned}\dot{e}_i(t) =& -c_ie_i(t) - (c_i(t) - c_i)y_i(t) + \sum_{j=1}^{n} a_{ij}g_j(e_j(t)) + \sum_{j=1}^{n}(a_{ij}(t) - a_{ij})f_j(y_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}g_j(e_j(t-\tau(t))) + \sum_{j=1}^{n}(b_{ij}(t) - b_{ij})f_j(y_j(t-\tau(t))) + l_i(t)e_i(t) + k_i(t)e_i(t-\tau(t)),\end{aligned} \tag{5}$$

另一种形式为，

$$\begin{aligned}\dot{e}(t) = &-Ce(t) - (C(t) - C)y(t) + A(t)g(e(t)) + (A(t) - A)f(y(t)) + Bg(e(t-\tau(t))) \\ &+ (B(t) - B)f(y(t-\tau(t))) + Le(t) + K(e(t-\tau(t))),\end{aligned} \tag{6}$$

这里 $g(e(t)) = f(x(t) + e(t)) - f(x(t))$。

贯穿整篇文章，我们有如下两个假设：

(H_1): 假设激活函数$f_i(x), i = 1, 2, \cdots, n$.满足如下的条件：存在常数$k_i > 0$，使得对于任意的$x_1, x_2 \in \mathbb{R}$, 有

$$|f_i(x_1) - f_i(x_2)| \leqslant k_i|x_1 - x_2|, i = 1, 2, \cdots, n.$$

(H_2): $\tau(t) \geqslant 0$ 是一个微分函数，且对于所有的t，有$0 \leqslant \dot{\tau}(t) \leqslant \mu < 1$ 。

定义1.1 系统(1) 和(3) 是在自适应控制器下是全局同步的，如果

$$\lim_{t\to\infty} \|x_i(t) - y_i(t)\| = 0, i = 1, 2, \cdots, n.$$

这里$\|\cdot\|$ 代表$\mathbb{R}^n$ 上的欧几里得范数。

引理1.1 对任何向量$a, b \in \mathbb{R}^n$，不等式

$$\pm 2a^{\mathrm{T}}Xb \leqslant a^{\mathrm{T}}Xa + b^{\mathrm{T}}Xb$$

满足，其中X 是任何矩阵，其中$X > 0$。

2. 主要结果

考虑以上的模型，在理论证明以后，我们可以得到以下结论。

定理2.1 让$c_i(t), a_{ij}(t), b_{ij}(t), l_i(t), k_i(t)(i,j=1,2,\cdots,n)$ 满足：

$$\begin{aligned}
&\dot{c}_i(t)=\gamma_i e_i(t)y_i(t)exp(\mu t)\\
&\dot{a}_{ij}(t)=-\eta_{ij}e_i(t)f_j(y_j(t))exp(\mu t)\\
&\dot{b}_{ij}(t)=-\rho_{ij}e_i(t)f_i(y_j(t-\tau(t)))exp(\mu t)\\
&\dot{l}_i(t)=-\varepsilon_i^2 e_i^2(t)exp(\mu t), \dot{k}_i(t)=-\sigma_i^2 e_i(t-\tau(t))e_i(t)exp(\mu t)
\end{aligned} \tag{7}$$

这里$e_i(t)=y_i(t)-x_i(t)(i=1,2,\cdots,n), \mu\geqslant 0$ 是一个正常数，并且$\gamma_i, \eta_{ij}, \rho_{ij}, \sigma_{ij}$ 都是随意的正常数，且

$$l=\lambda_{max}(-C)+\lambda_{max}(\frac{\mu}{2}E)+\lambda_{max}(\frac{1}{2}AA^{\mathrm{T}})+\lambda_{max}(\frac{1}{2}BB^{\mathrm{T}})+\frac{1}{2}h+\frac{h}{2(1-\lambda)}e^{\mu\tau^+}.$$

这里$\lambda_{max}(M)$ 表示对称矩阵M的最大特征值，τ^+ 表示$\tau(t)$ 的上确界。如果假设(H_1) 和(H_2) 满足，并且$l<0$,则从系统(2)与主系统(1)是全局同步的，并且

$$\lim_{t\to\infty}(c_i(t)-c_i)=\lim_{t\to\infty}(a_{ij}(t)-a_{ij})=\lim_{t\to\infty}(b_{ij}(t)-b_{ij})=0, i,j=1,2,\cdots,n.$$

证明 构造如下李雅普诺夫泛函

$$\begin{aligned}
V(t)=&\frac{1}{2}e^{\mathrm{T}}(t)e(t)exp(\mu t)+\frac{1}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}(c_i(t)-c_i)^2+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\eta_{ij}}(a_{ij}(t)-a_{ij})^2\\
&+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\rho_{ij}}(b_{ij}(t)-b_{ij})^2+\frac{1}{2}\sum_{i=1}^{n}\frac{l_i^2(t)}{\varepsilon_i^2}+\frac{1}{2}\sum_{i=1}^{n}\frac{k_i^2(t)}{\sigma_i^2}\\
&+\int_{t-\tau(t)}^{t}\frac{1}{2(1-\lambda)}g^{\mathrm{T}}[e(\theta)]g[e(\theta)]exp\left\{\mu[\theta+\tau(t)]\right\}d\theta
\end{aligned} \tag{8}$$

计算(8) 的导数，并代入误差系统(6),

$$\begin{aligned}
\dot{V}(t)=&e^{\mathrm{T}}(t)\dot{e}(t)exp(\mu t)+\frac{\mu}{2}e^{\mathrm{T}}(t)e(t)exp(\mu t)+\sum_{i=1}^{n}\frac{1}{\gamma_i}(c_i(t)-c_i)\dot{c}_i(t)+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\eta_{ij}}(a_{ij}(t)-a_{ij})\dot{a}_{ij}(t)\\
&+\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{\rho_{ij}}(b_{ij}(t)-b_{ij})\dot{b}_{ij}(t)+\sum_{i=1}^{n}\frac{l_i(t)}{\varepsilon_i^2}\dot{l}_i(t)+\sum_{i=1}^{n}\frac{k_i(t)}{\sigma_i^2}\dot{k}_i(t)+\frac{1}{2(1-\lambda)}g^{\mathrm{T}}[e(t)]g[e(t)]exp(\mu t)\\
&*exp(\mu\tau(t))-\frac{1-\dot{\tau}(t)}{2(1-\lambda)}g^{\mathrm{T}}\left\{e[t-\tau(t)]\right\}g\left\{e[t-\tau(t)]\right\}exp(\mu t)
\end{aligned}$$

注意到定理3.1中的假设和方程(5)，我们得到

$$
\begin{aligned}
\dot{V}(t) = exp(\mu t)\Big\{ & - e^{\mathrm{T}}(t)Ce(t) + \frac{\mu}{2}e^{\mathrm{T}}(t)e(t) + e^{\mathrm{T}}(t)Ag[e(t)] + e^{\mathrm{T}}(t)Bg[e(t-\tau(t))] \\
& + \frac{1}{2(1-\lambda)}g^{\mathrm{T}}[e(t)]g[e(t)]exp(\mu\tau(t)) - \frac{1-\dot{\tau}(t)}{2(1-\lambda)}g^{\mathrm{T}}\{e[t-\tau(t)]\}\,g\{e[t-\tau(t)]\}\Big\}
\end{aligned}
$$

注意到假设(H_2):$0 \leqslant \dot{\tau}(t) \leqslant \mu < 1$,我们可以得到$-[\frac{(1-\dot{\tau}(t))}{(1-\mu)}] \leqslant -1$.

通过引理1.1，将X用单位矩阵替代，我们有

$$
\begin{aligned}
\dot{V}(t) \leqslant exp(\mu t)\Big\{ & - e^{\mathrm{T}}(t)Ce(t) + \frac{\mu}{2}e^{\mathrm{T}}(t)e(t) + \frac{1}{2}e^{\mathrm{T}}(t)AA^{\mathrm{T}}e(t) \\
& + \frac{1}{2}g^{\mathrm{T}}[e(t)]\cdot g[e(t)] + \frac{1}{2}e^{\mathrm{T}}(t)BB^{\mathrm{T}}e(t) + \frac{1}{2}g^{\mathrm{T}}\{e[t-\tau(t)]\}\cdot g\{e[t-\tau(t)]\} \\
& + \frac{1}{2(1-\lambda)}g^{\mathrm{T}}[e(t)]g[e(t)]exp[\mu\tau(t)] - \frac{1-\dot{\tau}(t)}{2(1-\lambda)}g^{\mathrm{T}}\{e[t-\tau(t)]\}\cdot g\{e[t-\tau(t)]\}\Big\} \\
\leqslant exp(\mu t)\Big\{ & - e^{\mathrm{T}}(t)Ce(t) + \frac{\mu}{2}e^{\mathrm{T}}(t)e(t) + \frac{1}{2}e^{\mathrm{T}}(t)AA^{\mathrm{T}}e(t) + \frac{1}{2}g^{\mathrm{T}}[e(t)]\cdot g[e(t)] \\
& + \frac{1}{2}e^{\mathrm{T}}(t)BB^{\mathrm{T}}e(t) + \frac{1}{2(1-\lambda)}g^{\mathrm{T}}[e(t)]g[e(t)]exp[\mu\tau(t)]\Big\}
\end{aligned} \tag{9}
$$

注意到假设(H_1)，我们有如下的不等式:

$$
|f_i[e_i(t)]| \leqslant h_i|e_i(t)|, i = 1, 2, \cdots, n.
$$

然后，我们可以得到：

$$
g^{\mathrm{T}}[e(t)]\cdot g[e(t)] = \sum_{i=1}^{n} g_i^2[e_i(t)] \leqslant \sum_{i=1}^{n} h_i^2 e_i^2(t) \leqslant he^{\mathrm{T}}(t)e(t), \tag{10}
$$

这里$h = max\{h_i^2|i = 1, 2, \cdots, n\}$.

将不等式(10)代入不等式(9)的右边,

$$
\begin{aligned}
\dot{V}(t) \leqslant & exp(\mu t)\Big\{ - e^{\mathrm{T}}(t)Ce(t) + \frac{\mu}{2}e^{\mathrm{T}}(t)e(t) + \frac{1}{2}e^{\mathrm{T}}(t)AA^{\mathrm{T}}e(t) + \frac{1}{2}he^{\mathrm{T}}(t)e(t) \\
& + \frac{1}{2}e^{\mathrm{T}}(t)BB^{\mathrm{T}}e(t) + \frac{h}{2(1-\lambda)}e^{\mathrm{T}}(t)e(t)exp[\mu\tau(t)]\Big\} \\
& \leqslant exp(\mu t)e^{\mathrm{T}}(t)\Big\{\lambda_{max}(-C) + \frac{\mu}{2} + \lambda_{max}(\frac{1}{2}AA^{\mathrm{T}}) + \frac{1}{2}h + \lambda_{max}(\frac{1}{2}BB^{\mathrm{T}}) + \frac{h}{2(1-\lambda)}e^{\mu\tau^{+}}\Big\}\cdot e(t),
\end{aligned}
$$

最后，我们得到

$$
\dot{V}(t) \leqslant 0
$$

通过李雅普诺夫稳定性理论，我们可以知道从系统(2)与主系统(4)是全局同步的。与此同时，变系数$a_{ij}(t), c_i(t), b_{ij}(t)$ 将会同步于$a_{ij}, c_i, b_{ij}, i, j = 1, 2, \cdots, n$. 所以，我们完成了证明。

3. 数值模拟

下面我们将通过一个例子去验证定理2.1的有效性。

例子3.1 考虑带有时变时滞的耦合神经网络，用下列方程描述:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) + I \tag{11}$$

其中

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2.0 & -0.11 \\ -5.0 & 3.2 \end{pmatrix}$$
$$B = \begin{pmatrix} -1.6 & -0.1 \\ -0.18 & -2.4 \end{pmatrix}, \quad I = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

且$\tau(t) = \frac{e^t}{1+e^t}, f(t) = \tanh(t)$。神经网络(11)的初始值为$x_1(s) = -1.8, x_2(s) = 0.2, s \in [-1, 0]$。控制从系统为:

$$\dot{y}(t) = -C(t)y(t) + A(t)f(y(t)) + B(t)f(y(t-\tau(t))) + I + U(t) \tag{12}$$

其中，我们只确定八个参数$a_{11}(t), a_{22}(t), b_{11}(t), b_{22}(t), l_1(t), l_2(t), k_1(t), k_2(t)$。方便起见，我们将分别记为$a_1, a_2, b_1, b_2, l_1, l_2, k_1, k_2$。然后，我们必须构建如下反馈强度更新规律和参数自适应规律：

$$\begin{aligned}
\dot{a}_1 &= -9.2\,(y_3 - y_1) \tanh(y_3) \exp(0.5), \\
\dot{a}_2 &= -0.6\,(y_4 - y_2) \tanh(y_4) \exp(0.5), \\
\dot{b}_1 &= -9.0\,(y_3 - y_1) \tanh(y_3(t-\tau(t))) \exp(0.5), \\
\dot{b}_2 &= -0.8\,(y_4 - y_2) \tanh(y_4(t-\tau(t))) \exp(0.5), \\
\dot{l}_1 &= -5 \times 5y(5)y(5) \exp(0.5), \\
\dot{l}_1 &= -0.5 \times 0.5y(6)y(6) \exp(0.5), \\
\dot{k}_1 &= -3 \times 3\,(e_1(t-\tau(t)))\, y(5) \exp(0.5), \\
\dot{k}_1 &= -0.3 \times 0.3\,(e_2(t-\tau(t)))\, y(6) \exp(0.5),
\end{aligned}$$

各个初始值如下所示,

$$\begin{aligned}
&a_1(0) = -1.2, \quad a_2(0) = 5.8, \quad y_1 = 2.8, \quad y_2 = -0.2, \\
&b_1(0) = -0.2, \quad b_2(0) = 4.6, \quad l_1(0) = 5, \quad l_2(0) = 5, \\
&k_1(0) = 3, \quad k_2(0) = 3,
\end{aligned}$$

图 1展示了误差向量$e_1(t), e_2(t)$ 趋向于0；图 2 说明了随着时间进行，变系数趋于常数。

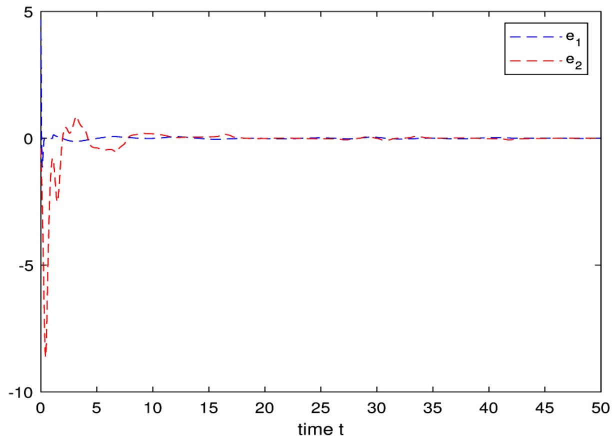

Figure 1. Evolution of synchronization errors $e_1(t), e_2(t)$, asymptotically achieve to 0

图 1. 同步误差$e_1(t), e_2(t)$趋于0

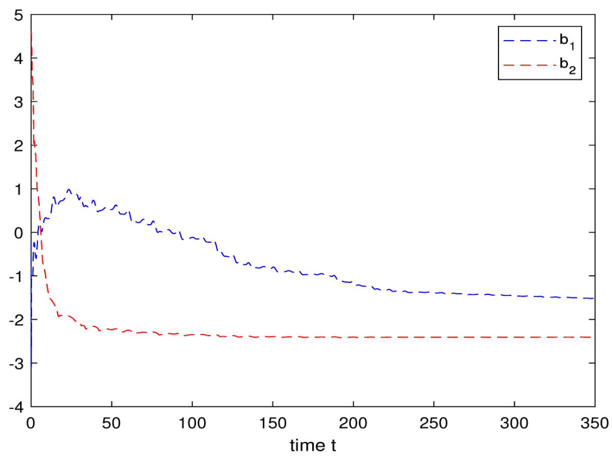

Figure 2. Variable coefficient b_1, b_2, asymptotically achieve to -1.6, -2.4

图 2. 变系数b_1, b_2分别趋向于-1.6, -2.4

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